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Prolongation of friction dominated evolution for superconducting cosmic strings

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Abstract: This investigation is concerned with cosmological scenarios based on particle physics theories that give rise to superconducting cosmic strings (whose subsequent evolution may produce stable loop configurations known as vortons). Cases in which electromagnetic coupling of the string current is absent or unimportant have been dealt with in previous work. The purpose of the present work is to provide quantitative estimates for cases in which electromagnetic interaction with the surrounding plasma significantly affects the string dynamics. In particular it will be shown that the current can become sufficiently strong for the initial period of friction dominated string motion to be substantially prolonged, which would entail a reinforcement of the short length scale end of the spectrum of the string distribution, with potentially observable cosmological implications if the friction dominated scenario lasts until the time of plasma recombination.

Keywords: Cosmological Phase Transitions, Physics of the Early Universe.

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Contents

1. Introduction

The original motivation for this work was the investigation of conditions under which it might be necessary to revise a previous analysis [1] of the vorton production that is likely to occur in the framework of various categories of "superconducting" cosmic string forming particle physics theories. This used a simplified picture in which the strings themselves arose as vortex defects of the vacuum formed by spontaneous symmetry breaking characterised by a mass scale $m_{\rm x}$ at a corresponding cosmological temperature

$$\Theta_{\rm x} \approx m_{\rm x}\,, \tag{1.1}$$

after which a (Witten type) "superconducting" current is supposed to condense on the string when the cosmological temperature Θ drops below another mass scale, $m_\sigma \lesssim m_{\rm x}$ (where for example $m_{\rm x}$ might be as high as the GUT scale, of the order of 10^{16} GeV, while m_σ might be as low as the electroweak scale, of the order of 10^2 GeV).

It has already been shown [2] that a revised estimate (according to which the vorton production is substantially enhanced) is required for cases in which the string is of the special chiral type [3], which can occur within the framework of many supersymmetric theories, but only when the relevant string current is of electromagnetically uncoupled kind. The present analysis was motivated by a question of a very different kind, concerning the kind of revision that might be relevant when the string current is of an electromagnetically coupled kind.

The preceding analysis [1] distinguished between two qualitatively different scenarios, depending on whether or not the current condensation temperature

$$\Theta_\sigma \approx m_\sigma \tag{1.2}$$

is sufficiently high to occur in the *friction dominated regime* that is characterised [4] by the condition

$$\tau \lesssim t \tag{1.3}$$

to the effect that the relevant friction damping time scale τ say should be short compared with the cosmological time scale $t \approx H^{-1}$, where, according to the standard Friedmann formula, the Hubble expansion rate H is given in terms of the mean cosmological mass-energy density ρ by $H^2 \simeq 8\pi G\rho/3$ where G is Newton's constant. Using units such that the speed of light c and the Dirac-Planck constant $\hbar$ are both set equal to unity, this cosmological time scale is given during the radiation dominated era (during which the transitions under consideration would have occurred) by an expression of the well-known form

$$t \approx \frac{m_{\rm P}}{\sqrt{\rho}}\,, \tag{1.4}$$

in which the mass energy density of the radiation is given in terms of the cosmological temperature Θ by

$$\rho \approx g^*\Theta^4\,, \tag{1.5}$$

where g^* is the effective number of massless degrees of freedom in the temperature range under consideration. Note that $g^* \approx 1$ at low temperatures but that in the range where vorton production is likely to occur, from electroweak unification through to grand unification, something more like $g^* \simeq 10^2$ would be a reasonable estimate.

The previous analysis [1] was based on the traditional supposition [5, 6] that the relevant friction damping time scale τ would have been given in terms of the string tension

$$\mathcal{T} \approx m_{\rm x}^2 \tag{1.6}$$

by an expression of the form

$$\tau \approx \frac{\mathcal{T}}{\beta\,\Theta^3}\,. \tag{1.7}$$

Here β is a dimensionless drag coefficient that was assumed to have an approximately constant value,

$$\beta \approx \beta_\sigma \tag{1.8}$$

depending on the details of the underlying field theory but typically estimated [6] to be of the order of unity,

$$\beta_\sigma \approx 1 \tag{1.9}$$

on the assumption that the dominant drag effect would be due to direct scattering of ambient thermal gas particles by the string core.

It has recently been pointed out by Dimopoulos and Davis [7] that if the string carried an electromagnetic current with sufficiently large local (root mean square) magnitude, I say, then this standard picture would need to be revised. The purpose of the present article is to demonstrate that there are many cases where the current will indeed become sufficiently large for such a revision to be needed, and to provide quantitative estimates showing how the traditional picture of string evolution will need to be modified in such cases.

It will be shown below that there are indeed many scenarios in which electromagnetic currents can engender cosmologically interesting consequences [8] by substantially modifying the evolution of the main part of the string distribution. However it would appear that the effect of such modifications will always be either too little or else too late to invalidate the estimates of vorton production [1] that were cited above. (It is however to be remarked that this confirmation of preceeding work on vorton production should not be considered to be definitive, since as well as the neglect of electromagnetic drag, which is shown here to be generally justifiable, this work [1] also involved other simplifying assumptions and approximations that still need further investigation.)

The outline of this paper is as follows: in the following section we discuss the new physical effect which leads to the changes in the cosmological evolution of the vorton density, namely plasma drag. It follows that if the current $I(t)$ becomes larger than the critical current I_c, the friction dominated regime of the cosmic string network will endure indefinitely. In section 3 we derive the expression for the *wiggle smoothing length* ξ, the crucial length scale of the string network which determines the density in strings. In section 4 we discuss the initial current on the strings. It is in this section that the crucial differences compared to the original analysis of Dimopoulos and Davis [7] arise. In section 5 we follow the evolution of the current during the string network evolution. An important parameter which enters the analysis at this point is the *blueshift factor* $\mathcal{Z}$. The value of this factor is derived in the subsequent section. In section 7, the results of the two previous sections are combined to derive the expression for the current $I(t)$ on the string. The case when $I(t)$ remains smaller than I_c is analyzed in section 8, the other case is discussed in section 9. Our results are summarized in section 10.

A word concerning the notation adopted in this paper: we work in units in which the both speed of light and Planck's constant are set to 1. Newton's constant G is not set equal to 1, and therefore the Planck mass m_p often arises in our formulas. Perhaps unconventionally, we denote the temperature by Θ and reserve the symbol $\mathcal{T}$ for the string tension. The cosmological time is denoted by t.

2. Plasma drag on an electrically conducting string

The reason why the traditional picture will need to be modified in cases when the string current becomes sufficiently high is the resulting enhancement of friction drag exerted on a relativistically moving cosmic string by its surrounding thermal background. The effect of the drag contribution that will be produced by electromagnetic interaction with the surrounding plasma, will be characterised by a friction damping time scale τ whose order of magnitude has been estimated [7] to be given by

$$\tau \approx \frac{\mathcal{T}}{I\sqrt{\rho}} . \tag{2.1}$$

The difference in the friction damping time arises because an electromagnetically coupled superconducting string creates a magnetic field around the string core. The field is of sufficient strength that it does not allow the plasma particles to approach the string, creating a magnetocylinder around the string, whose radius increases with the string current. This results in a much larger scattering cross-section than in the non-electromagnetically coupled case and hence a stronger frictional drag. This plasma friction damping formula (2.1) is consistent with the preceeding general formula (1.7) if, instead of using the constant (1.8), we take the dimensionless coefficient β to have a variable value given by the expression

$$\beta \approx \frac{\sqrt{g^*} I}{\Theta} , \tag{2.2}$$

which will be applicable whenever it gives a value large compared with the order of unit value given by (1.9). On the other hand in what we shall refer to as the low current regime, when (2.2) gives a result that is small compared with unity, it is the traditional formula, as given by (1.8), that will be applicable.

Whichever formula applies, i.e. whether or not the coefficient β is large compared with unity, it can be seen that it can be used to express the ratio of the cosmological time scale t to the damping time scale τ in the form

$$\frac{t}{\tau} \approx \frac{m_{\rm P}\, \beta\, \Theta}{\sqrt{g^*}\, m_{\rm x}^2} . \tag{2.3}$$

In the traditional picture characterised by (1.8), the friction dominated regime (1.3), for which the ratio (2.3) is large, comes to an end when the temperature Θ drops below the Kibble limit value $\Theta_\star$ given by

$$\Theta_\star \approx \frac{\sqrt{g^*}\,\Theta_{\rm x}^2}{\beta_\sigma\, m_{\rm P}} . \tag{2.4}$$

However in the plasma damping scenario characterised by (2.1) it can be seen from (1.4) that one will have

$$\frac{t}{\tau} \approx \frac{m_{\rm P}\, I}{m_{\rm x}^2} , \tag{2.5}$$

so it follows, as was observed by Dimopoulos and Davis, that if I remains greater than a certain critical lower limit value,

$$I \gtrsim I_{\rm c} , \tag{2.6}$$

where [7]

$$I_{\rm c} \approx \frac{m_{\rm x}^2}{m_{\rm P}} , \tag{2.7}$$

then according to (2.5) the inequality (1.3) will always be satisfied. Thus if (2.6) continued to hold, then the friction dominated regime would endure indefinitely (at least until the end of the radiation dominated era, when the foregoing assumptions will cease to be applicable). On the other hand whenever (2.6) is not satisfied, electromagnetic friction damping will be effectively negligible.

One of the main purposes of the present work is to investigate the conditions under which the current I actually can develop an amplitude exceeding the critical value (2.7) above which electromagnetic drag is important.

3. Evaluation of the wiggle smoothing length

Before proceeding to the estimation of the magnitude of the current amplitude I itself, it will be useful to consider how it influences the wiggle smoothing length scale ξ (denoted by R in the preceding work [7] and by $L_{\rm min}$ in the previous analysis [1]). In the friction dominated epoch, this scale is (roughly) defined as the minimum value for a length scale L for which string wiggles of significant amplitude are present, i.e. ξ is the length scale below which dissipative ironing out has had time to be effective. One of the reasons why this *extrinsic* wiggle smoothing length ξ (which should not be confused with the *internal* electromagnetic smoothing length ζ introduced below) is important is that it determines the string energy per unit volume, $\rho_{\rm s}$ say, which can be seen by elementary dimensional considerations [12, 1] to be given in order of magnitude by an expression of the form

$$\rho_{\rm s} \approx \mathcal{T} \nu \xi^{-2} , \tag{3.1}$$

where ν is a dimensionless parameter, that will remain approximately constant with a magnitude of order of unity so long as resistive damping is dominant, but that will acquire a much smaller value if and when radiative damping takes over as the dominant loss mechanism.

According to the generally accepted picture [12], during the friction dominated regime (which would last at least so long as the temperature exceeds the Kibble limit (2.4) even if electromagnetic damping were negligible) the string distribution will be of Brownian type on all length scales L above the relevant wiggle smoothing length ξ, which will be given in terms of the cosmological time t and the relevant friction damping time scale τ by

$$\xi \approx (\tau t)^{1/2} . \tag{3.2}$$

In these circumstances the factor ν in (3.1) will have a value given roughly by

$$\nu \approx \nu_\star , \tag{3.3}$$

where $\nu_\star$ is a constant of the order of unity, and it can be seen from (1.4) and (1.7) that we shall have

$$\xi^2 \approx \frac{m_{\rm x}^2 m_{\rm P}}{\sqrt{g^*}\beta\Theta^5} , \tag{3.4}$$

where β is given by (1.8) or (2.2) as the case may be.

In a radiation damping epoch, if there is one, the corresponding formula for the wiggle smoothing length scale will be given, according to the preceding analysis [1] (after a transition period[1] during which a scaling solution is established) by an expression of the form

$$\xi \approx \kappa t , \tag{3.5}$$

while the factor ν in (3.1) will be given by

$$\nu \approx \nu_\star \kappa^{3/2} , \tag{3.6}$$

where κ is a coefficient that will be given in terms of some order of unity efficiency factor Γ by

$$\kappa \approx \Gamma \left(\frac{m_{\rm x}}{m_{\rm P}} \right)^2 \tag{3.7}$$

when the dominant radiation loss mechanism is gravitational (as assumed in the preceeding analysis). Note that in the radiation dominated epoch the energy density is no longer determined by ξ, but instead by the inter-string separation.

[1] See [1, figure 1] for a sketch of the time evolution of ξ. During the transition period the motion of the strings is relativistic, leading the string separation to keep up with the Hubble radius. However, the wiggles on the strings cannot be erased until their scale falls below a new scale ξ set by the dominant energy loss mechanism.

The formula (3.7) would presumably have to be replaced by an expression of the analogous form

$$\kappa \approx \Gamma \left(\frac{I}{m_x}\right)^2 , \tag{3.8}$$

when the dominant radiation loss mechanism is electromagnetic , i.e. whenever the latter is larger. It is however to be observed that (assuming the efficiency factor Γ is comparable in the electromagnetic case to what it is in the gravitational case) the condition for the electromagnetic radiation damping coefficient (3.8) to dominate its gravitational analogue (3.7) will be the same as the condition (2.6) that is sufficient to guarantee friction dominance, at least so long as the temperature is above the Rydberg plasma recombination threshold. This means that in practice the formula (3.8) will never be needed before the recent low temperature (less than a few e.V.) matter dominated epoch. Throughout the preceeding radiation dominated epoch, which is what we are interested in here, it is the gravitational radiation damping formula (3.7) that will be relevant if and when friction damping becomes unimportant.

It is to be remarked that in a competition between radiation damping mechanisms the dominant one is that which gives the longest smoothing length. This contrasts with the situation in a competition between friction drag damping mechanisms, or between a friction drag mechanism and a radiating damping mechanism, for which the dominant mechanism will be that which gives the shortest smoothing length, due to the fact that the friction not only damps out the short wiggles but also effectively freezes in, and thus actively preserves, long wiggles.

4. Estimation of the initial current magnitude

Whereas the previous analysis [1] was based on the simplification of ignoring the electromagnetic (as opposed to inertial) effects of the relevant string currents, on the other hand the more recent work of Dimopoulos and Davis [7] was based on a no less drastic simplification according to which the relevant string current magnitude I was supposed to remain constant, with value comparable to Witten's predicted [9] upper limit, $I_{\rm max} \approx I_x$ with $I_{\rm max} \approx I_x$

$$I_x \approx e m_x \tag{4.1}$$

for a bosonic current, or the rather stricter upper limit $I_{\rm max} \approx I_\sigma$ with

$$I_\sigma \approx e m_\sigma \tag{4.2}$$

for a fermionic current, where $e \simeq 1/\sqrt{137}$ is the charge coupling constant.

The aim of the present work is to develop a more realistic intermediate picture, which not only allows for the effect of the electromagnetic coupling of the current but also allows for the dissipative damping of its amplitude.

What one expects, on dimensional grounds [1], is that at the time of its formation, when the cosmological temperature passes through the value (1.2) characterised by the relevant carrier mass m_σ, the current carrying field will be characterised by random fluctuations with wavelength

$$\lambda_\sigma \approx \Theta_\sigma^{-1}\,, \tag{4.3}$$

and that it will have a root mean square amplitude given by $I \approx I_\sigma$, which means that it will initially be close to saturation in the fermionic case — though not in the bosonic case if the Kibble mass scale $m_{\rm x}$ characterising the string itself is substantially larger than the carrier mass scale m_σ.

It is to be remarked that the ratio of this initial (and in the fermionic case maximal) current value I_σ to the critical value $I_{\rm c}$ given by (2.7), above which drag becomes important, will be given by

$$\frac{I_\sigma}{I_{\rm c}} \approx \sqrt{\frac{g^* e^2}{\beta_\sigma^2}}\left(\frac{\Theta_\sigma}{\Theta_\star}\right). \tag{4.4}$$

Since the numerical coefficient $\sqrt{g^* e^2/\beta_\sigma^2}$ will have an order of magnitude comparable with unity at most, this means that I_σ will fall short of the critical value $I_{\rm c}$ if Θ_σ is below the Kibble limit temperature $\Theta_\star$.

5. Evolution of the current magnitude

Due to the subsequent contraction of the string — which occurs despite the effect of cosmological expansion due to the various frictional (or at a later stage, radiative) wiggle damping mechanisms to which it is subjected — the comoving wavelength of the initial wavelength will gradually shrink to a time dependent value

$$\lambda = \mathcal{Z}\lambda_\sigma\,, \tag{5.1}$$

where (using the same notation as in the previous analysis [1]) $\mathcal{Z}$ is the relevant time dependent blueshift factor.

If this blueshifting were the only process going on, the local root mean square current amplitude would be amplified to a magnitude of the order of $I \approx e\lambda^{-1} \approx \mathcal{Z}^{-1} I_\sigma$, a value that needs to be distinguished from the mean value, $\langle I\rangle_{\{L\}}$, averaged over a macroscopic length scale L say, which will be obtainable (as the result of a random walk process) as

$$\langle I\rangle_{\{L\}} \approx \mathcal{Z}^{-1} I_\sigma \left(\frac{\lambda}{L}\right)^{1/2} \approx e(\lambda L)^{-1/2}\,. \tag{5.2}$$

It is this — much smaller — large scale mean value $\langle I\rangle$ that would be relevant for estimating the charge on a protovoton loop detaching with initial circumference L,

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but on the other hand, as Dimopoulos and Davis have emphasised it is the (larger) local root mean square current magnitude I that is relevant for the estimation of the frictional drag. Nevertheless it can not be expected to be quite so large as they assumed [7], because, despite the blue shifting of the relevant microscopic fluctuation length scale λ, the real value of the local root mean square current I will in practice become smaller than the preceeding estimate $I \approx e/\lambda$.

The reason why I will become small compared with $e\lambda^{-1}$ is that in the meanwhile, while dissipative shrinking characterised by the blueshift factor $\mathcal{Z}$ is taking place, other dissipation mechanisms will be damping the amplitude of the current fluctuations, with a characteristic time scale that will naturally be shortest for fluctuations with the shortest wavelengths. This means that the current fluctuations will end up by being effectively smoothed out on all length scales L below some critical smoothing length, ζ say, that will increase monotonically with time. Assuming the carrier current on the string is conserved, the effect of such smoothing will be to reduce the root mean square value I of the current amplitude from the undamped value e/λ, (where λ, as given by (5.1), is the adjusted wavelength of the initial fluctuations) to a magnitude identifiable with the average value $\langle I\rangle_{\{L\}}$ over the minimum undamped length scale $L \approx \zeta$. One thus obtains the estimate

$$I \approx \langle I\rangle_{\{\zeta\}} \approx \frac{e}{\sqrt{\lambda\zeta}} \,. \tag{5.3}$$

In a vacuum background the relevant dissipation mechanism would presumably just be electromagnetic radiation back-reaction, for which a crude dimensional estimate of the corresponding smoothing length ζ will be given by an expression of the form

$$\zeta \approx c_\zeta \, t \,, \tag{5.4}$$

where c_ζ is an electromagnetic smoothing speed that will be comparable with the speed of light, i.e. $c_\zeta \approx 1$. However the regime of interest here is that of the early universe, at temperatures large compared with the Rydberg ionisation energy, for which the relevant thermal background will be a plasma. In the context of a rather similar physical configuration in such a plasma background, it has been remarked by Sigl et al [10] that the main energy loss mechanism will arise from resistance to ambient electric charge displacements (attributable to Debye type screening oscillations) for which the relevant damping time scale τ_ζ will be given in terms of the corresponding wavelength, λ_ζ say, of the current fluctuations by an expression of the form

$$\tau_\zeta \approx \sigma_{\rm e} \lambda_\zeta^2 \,, \tag{5.5}$$

where $\sigma_{\rm e}$ is the electrical conductivity. A rough estimate [11] of the effective conductivity in such a cosmological plasma is provided by an expression of the form

$$\sigma_{\rm e} \approx \sqrt{g^*}\Theta \,, \tag{5.6}$$

in which it is to be remarked that the factor $\sqrt{g^*}$ is of the order of ten in most of the regime of interest, but that it drops to the order of unity after the temperature Θ falls below that of electroweak unification. Identifying the cut off ζ with the value of the wavelength λ_ζ for which the decay time scale τ becomes comparable with the age t of the universe, it can be seen to follow from (5.5) that its order of magnitude will be given by

$$\zeta^2 \approx \frac{t}{\sqrt{g^*}\Theta} . \tag{5.7}$$

This can be seen from (1.4) to be equivalent to taking the smoothing velocity in (5.4) to be given by

$$c_\zeta^2 \approx \frac{\Theta}{m_{\rm P}} , \tag{5.8}$$

so that one finally obtains an expression of the explicit form

$$\zeta \approx \sqrt{\frac{2\pi m_{\rm P}}{g^* \Theta^3}} \tag{5.9}$$

for the magnitude of the electromagnetic smoothing length itself.

Whichever dissipation mechanism is responsible, it can be seen by substituting (5.4) into (5.3) that the mean squared current magnitude will be given by an expression of the form

$$I^2 \approx \frac{e^2\sqrt{g^*}}{\mathcal{Z} c_\zeta} \left(\frac{m_\sigma}{m_{\rm P}}\right) \Theta^2 , \tag{5.10}$$

in which, for a vacuum background, one would simply have $c_\zeta \approx 1$, whereas in the plasma background of the radiation dominated regime of interest here the value of c_ζ will be given by (5.8) so that one obtains

$$\left(\frac{I}{I_\sigma}\right)^2 \approx \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}} \left(\frac{\Theta}{m_\sigma}\right)^{3/2} \frac{1}{\mathcal{Z}} . \tag{5.11}$$

The condition for the ordinary friction drag contribution, as characterised by the damping time scale (1.7) to be dominated by the electromagnetic drag contribution, as characterised by (2.1), is that the latter time scale should be shorter, or equivalently that the corresponding dimensionless coefficient (2.2) should be large compared to unity. This coefficient will be given by

$$\beta \approx b \sqrt{\frac{m_\sigma}{m_{\rm P}\, \mathcal{Z}}} , \tag{5.12}$$

where b is another positive dimensionless coefficient given by

$$b^2 = \frac{e^2 g^{*3/2}}{c_\zeta} . \tag{5.13}$$

It can be seen that (contrary to what would follow [7] if the current were supposed to retain a constant amplitude of the order of the fermionic saturation value I_σ) the electromagnetic friction damping contribution will remain negligible unless and until the blue shift factor gets below a limit given by

$$\mathcal{Z}\sqrt{\frac{\Theta}{m_\sigma}} \lesssim e^2 g^{*3/2}\sqrt{\frac{m_\sigma}{m_{\rm P}}}\,. \tag{5.14}$$

6. Estimation of the blue shift factor

In order to use the preceding formula to evaluate the root mean square current amplitude I as an explicit function of the cosmological temperature Θ it is necessary to know how the blue shift factor $\mathcal{Z}$ evolves as a function of the string mass per unit volume, $\rho_{\rm s}$ as given by (3.1). If the evolution were an entirely continuous process, the factor $\mathcal{Z}$ would simply be proportional to the total comoving string length,

$$\Sigma \approx \frac{\rho_{\rm s}}{\mathcal{T}\Theta^3}\,, \tag{6.1}$$

in a comoving volume characterised by the thermal length scale Θ^{-1}, but (as discussed in the previous work [1]) due to the fact that part of the string length will be lost in the form of small loops chopped off discontinuously at intersections, the blueshift factor will actually be given by an expression of the form

$$\mathcal{Z} \approx \left(\frac{\Sigma}{\Sigma_\sigma}\right)^{1-\varepsilon}\,, \tag{6.2}$$

where Σ_σ is the value of the string length Σ at time of the current condensation at temperature Θ_σ, and where the possibility of string loss into small loops is allowed for by the presence of the dimensionless index ε. This "loop formation efficiency factor" is difficult to estimate precisely, but can be expected to lie in the range $0 \leq \varepsilon \lesssim 1$.

Provided the condition (1.3) for friction dominance is satisfied — i.e. provided the temperature Θ exceeds the Kibble value $\Theta_\star$ given by (2.4) or the current exceeds the Dimopoulos-Davis limit (2.7) — it can be seen from (3.1) and (3.4) that we shall have

$$\Sigma \approx \frac{\nu_\star\sqrt{g^*}\beta\Theta^2}{m_{\rm P} m_{\rm X}^2}\,, \tag{6.3}$$

which in the electromagnetic drag dominated case can be seen by (2.2) to give

$$\Sigma \approx \frac{\nu_\star g^* I\Theta}{m_{\rm P} m_{\rm X}^2}\,. \tag{6.4}$$

Regardless of which drag mechanism is dominant, it follows that if the conditions for friction dominance were also satisfied at the time of the current condensation, i.e. if

$\Theta_\sigma \gtrsim \Theta_\star$, then (independently of the value of the dimensionless coefficient $\nu_\star$) the blueshift factor will be given by

$$\mathcal{Z} \approx \left(\frac{\beta\Theta^2}{\beta_\sigma\Theta_\sigma^2}\right)^{1-\varepsilon} . \tag{6.5}$$

In a radiation damping epoch, if there is one, the relevant formula for the wiggle smoothing length scale will be given, according to the preceding analysis [1] by an expression of the form (3.5), and the factor ν in (3.1)) will be given by (3.6) in which, as we have seen, the coefficient κ will be given by the gravitational damping formula (3.7) so that (after the transition period during which a scaling solution is established) we shall end up with

$$\Sigma \approx \frac{\nu_\star g^*\Theta}{\sqrt{\Gamma} m_{\rm P} m_{\rm x}} . \tag{6.6}$$

If the radiation damping epoch had already begun at the time of the current condensation, i.e. if $\Theta_\sigma \lesssim \Theta_\star$ then it can be seen that the blue shift factor will be given by

$$\mathcal{Z} \approx \left(\frac{\Theta}{\Theta_\sigma}\right)^{1-\varepsilon} . \tag{6.7}$$

On the other hand for $\Theta_\sigma \gtrsim \Theta_\star$ a subsequent radiation damping regime, if any, would be characterised by

$$\mathcal{Z} \approx \left(\frac{\sqrt{g^*} m_{\rm x}\Theta}{\sqrt{\Gamma}\beta_\sigma\Theta_\sigma^2}\right)^{1-\varepsilon} . \tag{6.8}$$

A remaining possibility that may be conceived — though we shall see that it will not actually occur in practise — is that for which the current condensation occurred for $\Theta_\sigma \lesssim \Theta_\star$, i.e. after the Kibble transition to a radiation damping regime characterised by (6.6), but for which the current subsequently became strong enough to restore a regime of friction dominance characterised by (6.3), in which we would evidently obtain

$$\mathcal{Z} \approx \left(\frac{\sqrt{\Gamma} I\Theta}{m_{\rm x}\Theta_\sigma}\right)^{1-\varepsilon} . \tag{6.9}$$

7. Initial low current regime.

Just after the current forming transition as the cosmological temperature Θ drops past the value $\Theta_\sigma \approx m_\sigma$, it can be seen that since, by its definition, the blue shift factor will still have unit magnitude, $\mathcal{Z} \approx 1$, at this stage, the immediate effect of the resistive dissipation will be to reduce the root mean square current amplitude I

from its initial condensation value I_σ as given by (4.1) to a considerably lower value which according to (5.11) will be given by

$$\left(\frac{I}{I_\sigma}\right)^2 \approx \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}}\,. \tag{7.1}$$

Since our investigation is concerned only with phase transitions occurring at or below the GUT level we shall have

$$m_\sigma \lesssim \frac{m_{\rm P}}{g^*}\,, \tag{7.2}$$

from which it can immediately be seen that (7.1) implies

$$I \lesssim I_\sigma\,, \tag{7.3}$$

and hence that the damped root mean square value I of the current amplitude will be below the saturation value $I_{\rm max}$ not only in the fermionic case (4.2) but also, a fortiori, in the bosonic case (4.1). Furthermore, since, in the usual kinds of GUT theory (though not of course in Hagedorn type models, which are not considered here) the order of magnitude of the effective number g^* of degrees of freedom in the relevant temperature range will never exceed the inverse fine structure constant $1/e^2 \simeq 137$, i.e. we shall have

$$g^* e^2 \lesssim 1\,, \tag{7.4}$$

it can be seen that (7.1) also implies

$$g^* I^2 \lesssim m_\sigma^2\,, \tag{7.5}$$

which means that the r.h.s. of (2.2) will be small compared with unity, and therefore that at this stage the electromagnetic drag contribution will be unimportant. Thus to begin with, after the condensation of the carrier particles characterised by the mass scale m_σ, there will a phase during which (as assumed in nearly all work prior to that of Davis and Dimopoulos [7]) the system will be in what we refer to as a low current regime, which is characterised by an order of unity drag coefficient

$$\beta \approx 1\,. \tag{7.6}$$

So long as this low current phase lasts, provided that the temperature has not yet dropped below the Kibble limit value (2.4), the relevant blue shift factor will be given according to (6.5) simply by

$$\mathcal{Z} \approx \left(\frac{\Theta^2}{m_\sigma^2}\right)^{1-\varepsilon}\,. \tag{7.7}$$

It therefore follows from (5.11) that the relevant mean squared current will be given as a rather slowly varying function of the cosmological temperature Θ by an expression of the form

$$\left(\frac{I}{I_\sigma}\right)^2 \approx \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}}\left(\frac{m_\sigma}{\Theta}\right)^{(1-4\varepsilon)/2}\,. \tag{7.8}$$

It seems plausible to suppose that the (not yet very well understood) value of the small dimensionless coefficient ε would be less than 1/4 in any regime of friction damping (though it might well be higher in a regime of radiation reaction damping, during which a lot of loop creation by string intersections might be expected) but even if ε were somewhat larger (in which case (7.8) would imply that the absolute value of I would actually undergo a slight decrease as Θ goes down) the r.h.s. of (2.2), which is proportional to relative value I/Θ, would still increase as Θ goes down. This means that (unlike what was supposed in nearly all earlier work) the low current regime will not necessarily last indefinitely, but may be brought to an end when the cosmological temperature Θ has fallen to a critical value $\Theta_{\rm E}$ say, below which the main friction contribution will be of electromagnetic origin. This will occur when the r.h.s. of (2.2) reaches the order of unity, so it can be seen that the relevant electromagnetic transition temperature $\Theta_{\rm E}$ will be given by

$$\left(\frac{\Theta_{\rm E}}{m_\sigma}\right)^{5-4\varepsilon} \approx \frac{g^{*3} e^4 m_\sigma}{m_{\rm P}}\,, \tag{7.9}$$

provided that this value of $\Theta_{\rm E}$ is still above the Kibble limit Θ_σ given by (2.4). This necessary condition for a transition to an electromagnetic friction drag regime can be seen to be expressible as

$$\left(\frac{\Theta_\sigma}{\Theta_\star}\right)^{5-4\varepsilon} \gtrsim \frac{m_{\rm P}}{g^{*3} e^4 m_\sigma}\,, \tag{7.10}$$

or equivalently

$$\left(\frac{\sqrt{g^*} m_{\rm x}}{\beta_\sigma m_{\rm P}}\right)^{2-2\varepsilon} \lesssim \sqrt{\beta_\sigma} e^2 (g^*)^{5/4} \left(\frac{m_\sigma}{m_{\rm x}}\right)^{3-2\varepsilon}\,. \tag{7.11}$$

This condition only excludes the cases of extreme disparity for which the strings formation energy scale $m_{\rm x}$ is comparable with the GUT level but the current condensation energy scale m_σ is way down nearer the electroweak level. The condition (7.11) for the occurrence of an electromagnetic friction dominated regime will be satisfied in most other kinds of scenario, in which either both the string forming and the current forming phase transitions occur near the GUT level or else they both occur at much lower levels.

8. Final low current regime

If the condition (7.10) is not satisfied, then when the temperature has fallen to the Kibble limit value Θ_σ given by (2.4) it will enter a radiation damping regime in which the blue shift factor $\mathcal{Z}$ will no longer be governed by (7.7). If, as supposed above, the current condensation phase transition had already occurred at a temperature Θ_σ

higher than the Kibble limit value $\Theta_\star$, then $\mathcal{Z}$ will be governed by (6.8) so by (5.11) we shall obtain

$$\left(\frac{I}{I_\sigma}\right)^2 \approx \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}} \left(\frac{\sqrt{\Gamma}\beta_\sigma m_\sigma}{\sqrt{g^*} m_{\rm x}}\right)^{1-\varepsilon} \left(\frac{\Theta}{m_\sigma}\right)^{(1+2\varepsilon)/2} . \tag{8.1}$$

This evidently implies that the current amplitude I will continue to decrease as the temperature goes down, so there will be no possibility of it building up again to a sufficiently high value for friction drag to become important again. The implication of this is that this final low current radiation drag dominated regime will last indefinitely.

A similar conclusion, namely that electromagnetic drag will never become important, will be obtained a fortiori in the case for which the system is already in the radiation damping regime below the Kibble limit $\Theta_\star$ when the phase transition at the temperature Θ_σ occurs. In this case the initial low current stage described in the preceeding section will be skipped, and the system will pass directly to a final low current regime in which, instead of by (6.8), the blueshift factor will be governed by (6.7), so that instead of (8.1) the equation governing the decay of the current amplitude will have the simpler form

$$\left(\frac{I}{I_\sigma}\right)^2 \approx \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}} (\Theta m_\sigma)^{(1+2\varepsilon)/2} . \tag{8.2}$$

9. Current dominated regime

The main innovation in the present work is the investigation of cases for which the condition (7.10) is satisfied, i.e. for which we have

$$\Theta_{\rm E} \gtrsim \Theta_\star , \tag{9.1}$$

so that after the transition past the threshold (7.9) when the temperature becomes low enough to satisfy

$$\Theta \leq \Theta_{\rm E} , \tag{9.2}$$

there will occur what we shall refer to as a current dominated regime, meaning a regime in which the current amplitude I is large enough for electromagnetic friction drag to provide the main dissipation mechanism acting on the cosmic string distribution. A noteworthy feature of such a regime is that the ratio of the string mass density $\rho_{\rm s}$ as given by (3.1)[2] to the total mass density ρ as given by (1.5) will be expressible, using (2.2) and (3.4), by

$$\frac{\rho_{\rm s}}{\rho} \approx \nu_\star \frac{I}{m_{\rm P}} , \tag{9.3}$$

[2]Note that in this case there is never a radiation-dominated epoch so that the use of (3.1) is justified.

(in which it is to be recalled that $\nu_\star$ is just a constant of the order of unity). Such a regime can only last so long as I remains above the Dimopoulos-Davis critical value $I_{\rm c}$ given by (2.6), a condition which can be seen from (9.3) to be expressible as

$$\frac{\rho_{\rm s}}{\rho} \gtrsim \nu_\star \left(\frac{m_{\rm x}}{m_{\rm P}}\right)^2 . \tag{9.4}$$

This condition ensures the preservation of wiggle structure on length scales L down to a minimum smoothing value ξ given according to (2.5) and (3.2) by

$$\xi^2 \approx \frac{I_{\rm c}}{I} t^2 , \tag{9.5}$$

that satisfies the condition of remaining short

$$\xi \lesssim t , \tag{9.6}$$

compared with the cosmological length scale t.

Under these conditions, the comoving string length Σ will be given by (6.4) so by (6.2) the redshift factor will be given by

$$\mathcal{Z} \approx \left(\frac{\sqrt{g^*} e I \Theta}{\beta_\sigma I_\sigma \Theta_\sigma}\right)^{1-\varepsilon} . \tag{9.7}$$

However, unlike the corresponding formula (7.7) for the preceding low current regime, the expression (9.7) does not, by itself, specify the explicit temperature dependence of $\mathcal{Z}$, because it also involves the current amplitude I, whose evolution, according to (5.11) is also $\mathcal{Z}$ dependent. To obtain the explicit temperature dependence of these quantities, we start by eliminating $\mathcal{Z}$ between (5.11) and (9.7) so as to obtain the relation

$$\left(\frac{I}{I_\sigma}\right)^{3-\varepsilon} \approx \left(\frac{\beta_\sigma}{\sqrt{g^*} e}\right)^{1-\varepsilon} \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}} \left(\frac{\Theta}{m_\sigma}\right)^{(1+2\varepsilon)/2} . \tag{9.8}$$

The corresponding value for $\mathcal{Z}$ will be given by

$$\mathcal{Z}^{3-\varepsilon} \approx \left(\frac{g^* e^2}{\beta_\sigma^2} \sqrt{\frac{g^* m_\sigma}{m_{\rm P}}} \left(\frac{\Theta}{m_\sigma}\right)^{7/2}\right)^{1-\varepsilon} . \tag{9.9}$$

Since one would expect the loop formation efficiency coefficient ε to be small in a friction dominated regime, the preceeding formula implies that the temperature dependence of I will be rather weak, but nevertheless it can be seen that in the long run the current amplitude I will decrease as the temperature goes down. This means that there will be no chance of reaching a current saturation regime such as was originally envisaged by Dimopoulos and Davis [7]. However it does not exclude the likelihood that the friction dominated regime may be considerably prolonged.

In the very long run of course, the friction dominated regime must ultimately be terminated by a transition to the usual kind of radiation damping dominated regime, but if the relevant mass scales are relatively low (compared with the G.U.T, level) this final transition might not occur until by passage through the plasma recombination when the temperature reaches the Rydberg level,

$$\Theta_{\rm R} \approx e^4 m_{\rm e}\,, \tag{9.10}$$

(where $m_{\rm e}$ is the electron mass) but otherwise it will occur at a higher temperature when I gets down to value $I_{\rm c}$ given by (2.7). It can be seen from (9.8) that this will occur at a critical temperature $\Theta_{\rm c}$ say given by

$$\left(\frac{\Theta_{\rm c}}{m_\sigma}\right)^{(1+2\varepsilon)/2} \approx \left(\frac{\sqrt{g^*}}{\beta_\sigma}\right)^{1-\varepsilon} \sqrt{\frac{m_{\rm P}}{g^* e^4 m_\sigma}} \left(\frac{m_{\rm x}^2}{m_{\rm P} m_\sigma}\right)^{3-\varepsilon}. \tag{9.11}$$

After passing below this critical temperature $T_{\rm c}$, the universe will find itself in a final low current radiation dominated state of the kind described in the preceeding section, with the current magnitude governed by (8.1).

10. Conclusions

The preceeding analysis shows that the occurrence of an electromagnetic friction dominated regime will not have any effect on the vorton production estimates whose verification provided the original motivation for this work.[3] To understand this result, it is important to recall from [1] that in the friction-dominated case the vorton density is dominated by those produced at the onset of the friction-dominated period. From our analysis it has transpired that such an electromagnetic friction dominated regime only occurs if the electromagnetic current condensation Θ_σ is higher than the Kibble limit temperature $\Theta_\star$, and in that case (though not in general) the dominant vorton production mechanism [1] operates immediately and is unaffected by the subsequent evolution (friction dominated or otherwise) of the main part of the string distribution.

The fact that it does not affect our present (provisional) understanding of vorton formation does not exclude the possibility that the effect of electromagnetic friction drag on strings may be cosmologically important. In particular — while the preceeding analysis implies that its occurrence may be more delicately parameter dependent than was suggested in when it was originally envisaged by Dimopoulos and Davis [7] — the possibility evoked in the preceeding section that the friction drag dominated epoch may continue until the stage of plasma recombination has potentially interesting observational implications in view of the fact that this "last scattering" period is directly accessible as the source of the cosmic thermal background radiation. This will occur if

$$\Theta_{\rm c} \lesssim \Theta_{\rm R}\,, \tag{10.1}$$

[3] A similar negative conclusion has been reached in unpublished work by Dimopoulos [13]

where Θ_c is given by (9.11) and Θ_R by (9.10), or to be more explicit if

$$\left(\frac{m_x}{m_P}\right)^3 \left(\frac{m_x}{m_\sigma}\right)^{3-2\varepsilon} \lesssim e^2 \beta_\sigma \left(\frac{\sqrt{g^*}}{\beta_\sigma}\right)^{\varepsilon} \left(\frac{e^4 m_e}{m_P}\right)^{(1+2\varepsilon)/2} . \tag{10.2}$$

Despite the small value of the Rydberg to Planck energy ratio $e^4 m_e/m_P \approx 10^{-26}$ it can be seen that (because of the high powers of the factors on the left-hand side) in cases for which $m_\sigma \approx m_x$ this condition can easily be satisfied for values of the string formation energy scale m_x that can be very large compared with the electroweak level $m_\sigma/m_P \approx 10^{-16}$, though not if it is too near the GUT level $m_\sigma/m_P \approx 10^{-3}$ In such a case the spectrum of the string distribution at the time of electromagnetic decoupling would be characterised by the preservation of a higher proportion of short wavelength structure than in the traditional radiation dominated scenario, and it can be seen from the inequality (9.4) that the corresponding string mass density fraction will be higher than the traditionally predicted value $\rho_s/\rho \approx (m_x/m_P)^2$.

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